Date : June 2017

Review: June 2019

Aldbury C of E Primary School

Written
Calculation Policy

June 2017

Rationale

This policy outlines a model progression through written strategies for addition, subtraction, multiplication and division in line with the National Curriculum through the policy, we aim to link key manipulatives and representations in order that the children can be vertically accelerated through each strand of calculation. We know that school wide policies, such as this, can ensure consistency of approach, enabling children to progress stage by stage through models and representations they recognise from previous teaching, allowing for deeper conceptual understanding and fluency. As children move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding. However, it is expected that the majority of children in each class will be working at age-appropriate levels as set out in the National Curriculum and in line with Aldbury School policy.

The importance of mental mathematics

While this policy focuses on written calculations in mathematics, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. The following checklists outline the key skills and number facts that children are expected to develop throughout the school.

To add and subtract successfully, children should be able to:

- recall all addition pairs to 9 + 9 and number bonds to 10
- recognise addition and subtraction as inverse operations
- add mentally a series of one digit numbers (e.g. 5 + 8 + 4)
- add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. 600 + 700, 160 — 70)
- partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 74 into 70 + 4 or 60 + 14)
- use estimation by rounding to check answers are reasonable

To multiply and divide successfully, children should be able to:

- add and subtract accurately and efficiently
- recall multiplication facts to 12 x 12 = 144 and division facts to 144 ÷ 12 = 12
- use multiplication and division facts to estimate how many times one number divides into another etc.
- know the outcome of multiplying by 0 and by 1 and of dividing by 1
- understand the effect of multiplying and dividing whole numbers by 10, 100 and later 1000
- recognise factor pairs of numbers (e.g. that 15 = 3 x 5, or that 40 = 10 x 4) and increasingly able to recognise common factors
- derive other results from multiplication and division facts and multiplication and division by 10 or 100 (and later 1000)
- notice and recall with increasing fluency inverse facts
- partition numbers into 100s, 10s and 1s or multiple groupings
- understand how the principles of commutative, associative and distributive laws apply or do not apply to multiplication and division
- understand the effects of scaling by whole numbers and decimal numbers or fractions
- understand correspondence where n objects are related to m objects

- investigate and learn rules for divisibility

Progression in addition and subtraction

Addition and subtraction are connected.

<table>
<tr><td>Part</td><td>Part</td></tr>
<tr><td colspan="2">Whole</td></tr>
</table>

Addition names the whole in terms of the parts and **subtraction** names a missing part of the whole.

Progression in Multiplication and Division

Multiplication and division are connected.
Both express the relationship between a number of equal parts and the whole.

<table>
<tr><td>Part</td><td>Part</td><td>Part</td><td>Part</td></tr>
<tr><td colspan="4">Whole</td></tr>
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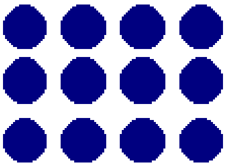


The following array, consisting of four columns and three rows, could be used to represent the number sentences: -

3 x 4 = 12, 4 x 3 =12, 3 + 3 + 3 + 3 = 12, 4 + 4 + 4 =12.

And it is also a model for division

$12 \div 4 = 3$

$12 \div 3 = 4$

$12 - 4 - 4 - 4 = 0$

$12 - 3 - 3 - 3 - 3 = 0$

Addition and Subtraction

Year 1

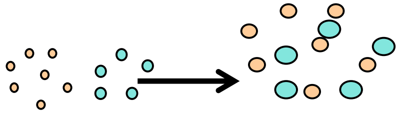
Numbers up to 100 through a wide variety of models and representations

These concepts are reinforced during Big Maths lessons, where children learn written methods before progressing to fully mental methods. addition, steps 5-12

Subtraction steps 5-12

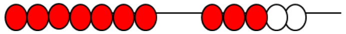
Addition

Combining two sets (aggregation)
Putting together – two or more amounts or numbers

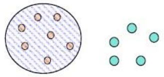


are put together to make a total
7 + 5 = 12

Count one set, then the other set. Combine the sets and count again. Starting at 1.
Counting along the bead bar, count out the 2 sets, then draw them together, count again. Starting at 1.



Combining two sets (augmentation)
This stage is essential in starting children to calculate rather than counting
Where one quantity is increased by some amount. Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number. Counters:



Start with 7, then count on 8, 9, 10, 11, 12

Number tracks:

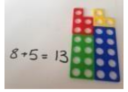
Start on 5 then count on 3 more

Different, manipulatives are combined to find a total, including but not limited to:

Multilink Towers:

Numicon

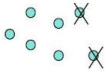
Number bonds to ten.

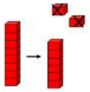


Combining two numbers then converting to show in tens and ones.

Subtraction

Taking away (separation model)
Where one quantity is taken away from another to calculate what is left. 7 – 2 = 5

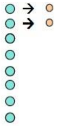




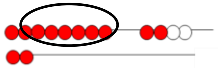
Multilink towers - to physically take away objects.

Numicon – physically taking three from 10

Finding the difference (comparison model)
Two quantities are compared to find the difference.
8 – 2 = 6
Counters:



Bead strings:



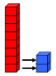
Make a set of 8 and a set of 2. Then count the gap.

Number tracks:

Start with the smaller number and count the gap to the larger number.

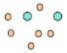
Different, manipulatives are compared to find a difference, including but not limited to:

Multilink Towers:



Numicon

1 set within another (part-whole model) The quantity in the whole set and one part are known, and may be used to find out how many are in the unknown part. 8 – 2 = 6 Counters:



Year 2

Numbers with up to and beyond 3 digits (read and write numbers up to at least 100 in numerals and words)

These concepts are reinforced during Big Maths lessons, where children learn written methods before progressing to fully mental methods. addition, steps 13-24

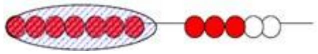
Subtraction steps 13-27

Addition

Bridging through 10s

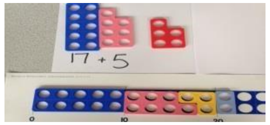
This stage encourages children to become more efficient and begin to employ known facts.

Bead string:



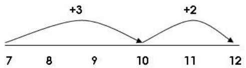
7 + 5 is decomposed / partitioned into 7 + 3 + 2. The bead string illustrates 'how many more to the next multiple of 10?' (children should identify how their number bonds are being applied) and then 'if we have used 3 of the 5 to get to 10, how many more do we need to add on? (ability to decompose/partition all numbers applied)

Number track:



Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Number line



Compensation model (adding 9 and 11) *This model of calculation encourages efficiency and application of known facts (how to add ten)*

7 + 9

Bead string:

Children find 7, then add on 10 and then adjust by removing 1.

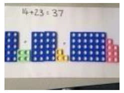
Working with larger numbers

Tens and ones + tens and ones *Ensure that the children have been transitioned onto Base 10 equipment and understand the abstract nature of the single 'tens' sticks and 'hundreds' blocks*

Partitioning (Aggregation model)

14 + 23 = 37

Numicon:

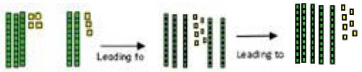


Children create the two sets with numicon equipment and then combine; ones with ones, tens with tens.

Partitioning (Augmentation model)

Base 10 equipment:

Encourage the children to begin counting from the first set of ones and tens, avoiding counting from 1. Beginning with the ones in preparation for formal columnar method.



Number line:

At this stage, children can begin to use an informal method to support, record and explain their method as learnt within Big Maths.

Subtraction

Bridging through 10s

This stage encourages children to become more efficient and begin to employ known facts.

Bead string:

12 – 7 is decomposed / partitioned in 12 – 2 – 5. The bead string illustrates 'from 12 how many to the last/previous multiple of 10?' and then 'if we have used 2 of the 7 we need to subtract, how many more do we need to count back? (ability to decompose/partition all numbers applied)

Number Track:

Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Number Line

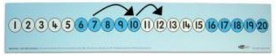
Counting up or 'Shop keepers' method

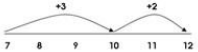
Bead string:

12 – 7 becomes 7 + 3 + 2.

Starting from 7 on the bead string 'how many more to the next multiple of 10?' (children should recognise how their number bonds are being applied), 'how many more to get to 12?'.

Number Track:





Compensation model (adding 9 and 11) (*This model of calculation encourages efficiency and application of known facts (how to add ten)*

18 – 9

Bead string

Children find 18, then subtract 10 and then adjust by adding 1.

Working with larger numbers

Tens and ones + tens and ones *Ensure that the children have been transitioned onto Base 10 equipment and understand the abstract nature of the single 'tens' sticks and 'hundreds' blocks*

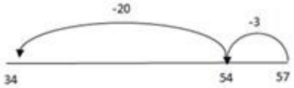
Take away (Separation model)

57 – 23 = 34

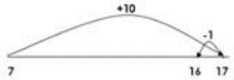
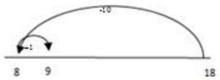
Base 10 equipment:

Children remove the lower quantity from the larger set, starting with the ones and then the tens. In preparation for formal decomposition.

Number Line:



At this stage, children can begin to use an informal method to support, record and explain their method as learnt within Big Maths.

	Number line:  		Number line:  	

Year 3

Numbers with up to at least 3 digits (whole numbers and decimals with up to 1 dp) through a wide variety of models and representations

These concepts are reinforced during Big Maths lessons, where children learn written methods before progressing to fully mental methods. addition, steps 25 - 28

Subtraction steps

28-29

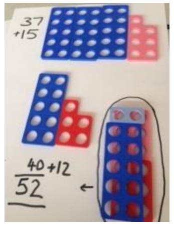
Addition

Bridging with larger numbers

Once secure in partitioning for addition, children begin to explore exchanging. What happens if the ones are greater than 10? Introduce the term 'exchange'. Using manipulatives, children exchange ten ones for a single tens rod, which is equivalent to crossing the tens boundary on the bead string or number line.

Numicon:

37 + 15 = 52

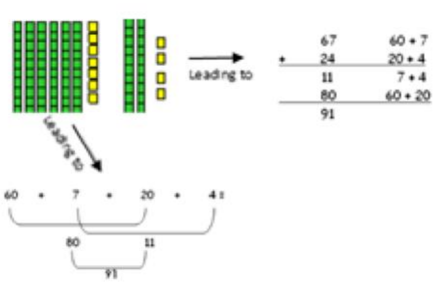


Discuss counting on from the larger number irrespective of the order of the calculation.

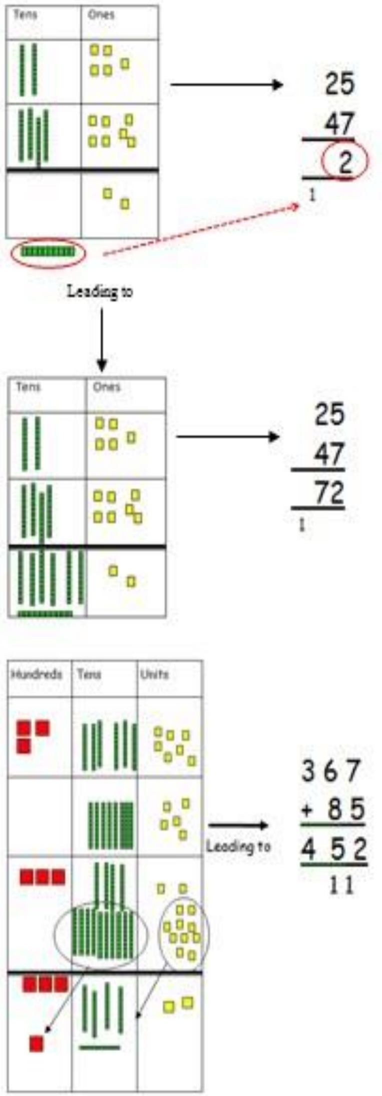
Expanded Vertical Method *Children are then introduced to the expanded vertical method to ensure that they make the link between using Base 10 equipment, partitioning and recording using this expanded vertical method.*

Base 10 equipment:

67 + 24 = 91



Compact method



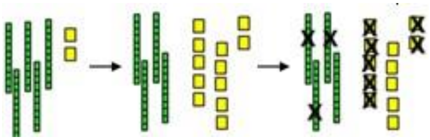
Subtraction

Bridging with larger numbers

Once secure in partitioning for addition, children begin to explore exchanging. What happens if the ones are greater than 10? Introduce the term 'exchange'. Using the Base 10 equipment, children exchange ten ones for a single tens rod, which is equivalent to crossing the tens boundary on the bead string or number line.

Base 10 equipment:

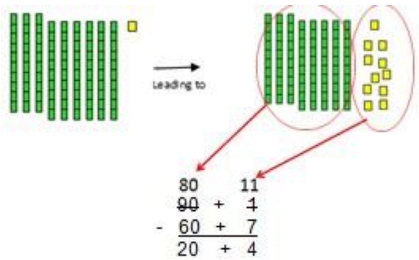
52-37= 15



Expanded Vertical Method *Children are then introduced to the expanded vertical method to ensure that they make the link between using Base 10 equipment, partitioning and recording using this expanded vertical method.*

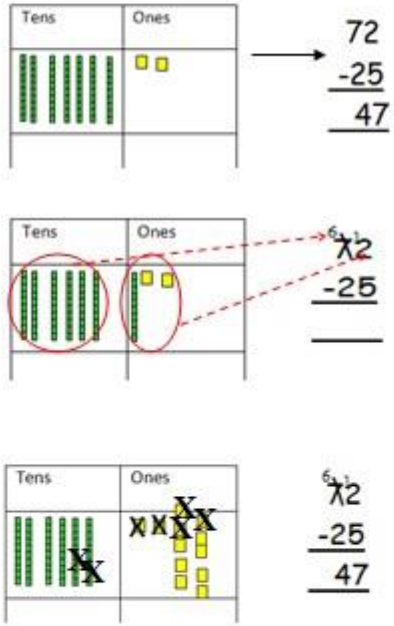
Base 10 equipment:

91 – 67 = 24



Compact decomposition

b



Years 46

Numbers up to and including 4 digits (whole numbers and decimal numbers with up to 2 decimal places) through a wide variety of models and representations

These concepts are reinforced during Big Maths lessons, where children learn written methods before progressing to fully mental methods. addition, steps 28 - 31
Subtraction steps 29-30

Numbers up to 1 million (whole numbers and decimal with up to 3 decimal places) through a wide variety of models and representations
Big maths addition, steps 32-38
Subtraction steps 31-36

Numbers up to 10 million (whole numbers, negative numbers and decimals with up to 3 decimal places) through a wide variety of models and representations

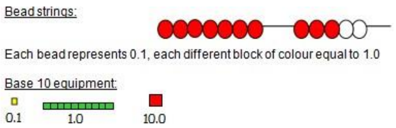
Big maths addition, steps 39-41
Subtraction steps 37

Addition

By returning to earlier manipulative experiences children are supported to make links across mathematics, encouraging 'If I know this…then I also know…' thinking.

Decimals

Ensure that children are confident in counting forwards and backwards in decimals – using bead strings to support.



Bead strings:

Each bead represents 0.1, each different block of colour equal to 1.0

Base 10 equipment:

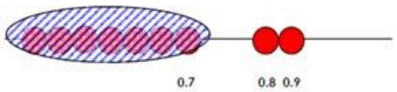
Addition of decimals Aggregation model of addition Counting both sets – starting at zero.

0.7 + 0.2 = 0.9

Augmentation model of addition

Starting from the first set total, count on to the end of the second set.

0.7 + 0.2 = 0.9

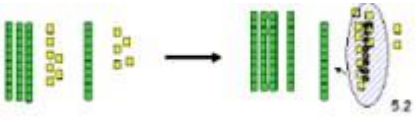


Bridging through 1.0

Encouraging connections with number bonds. 0.7 + 0.5 = 1.2

Partitioning

3.7 + 1.5 = 5.2



Column addition

789 + 642 becomes

$$\begin{array}{cccc} & 7 & 8 & 9 \\ + & 6 & 4 & 2 \\ \hline 1 & 4 & 3 & 1 \\ \hline & 1 & 1 & \end{array}$$

Gradation of difficulty- addition:

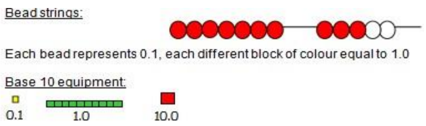
1. No exchange
2. Extra digit in the answer
3. Exchanging ones to tens
4. Exchanging tens to hundreds
5. Exchanging ones to tens and tens to hundreds
6. More than two numbers in calculation
7. As 6 but with different number of digits
8. Decimals up to 2 decimal places (same number of decimal places)
9. Add two or more decimals with a range of decimal places

Subtraction

By returning to earlier manipulative experiences children are supported to make links across mathematics, encouraging 'If I know this…then I also know…' thinking.

Decimals

Ensure that children are confident in counting forwards and backwards in decimals – using bead strings to support.



Bead strings:

Each bead represents 0.1, each different block of colour equal to 1.0

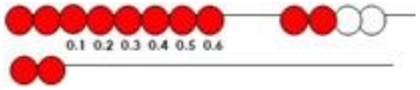
Base 10 equipment:

Subtraction of decimals
Take away model

0.9 – 0.2 = 0.7

Finding the difference (or comparison model):

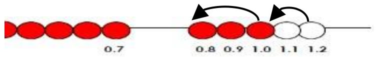
0.8 – 0.2 =



Bridging through 1.0

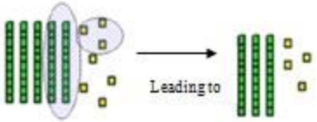
Encourage efficient partitioning.

1.2 – 0.5 = 1.2 – 0.2 – 0.3 = 0.7



Partitioning

5.7 – 2.3 = 3.4



Column subtraction

932 – 457 becomes

$$\begin{array}{cccc} & \overset{8}{\cancel{9}} & \overset{12}{\cancel{3}} & \overset{1}{2} \\ - & 4 & 5 & 7 \\ \hline & 4 & 7 & 5 \\ \hline \end{array}$$

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Gradation of difficulty- subtraction:

1. No exchange
2. Fewer digits in the answer
3. Exchanging tens for ones
4. Exchanging hundreds for tens
5. Exchanging hundreds to tens and tens to ones
6. As 5 but with different number of digits
7. Decimals up to 2 decimal places (same number of decimal places)
8. Subtract two or more decimals with a range of decimal places

Multiplication and Division

Year 1

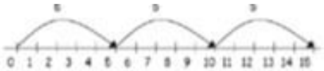
Counts in multiples of twos, fives and tens

These concepts are reinforced during Big Maths lessons, where children learn written methods before progressing to fully mental methods.

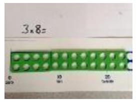
Multiplication

Early experiences
Children will have real, practical experiences of handling equal groups of objects and counting in 2s, 10s and 5s. Children work on practical problem solving activities involving equal sets or groups.

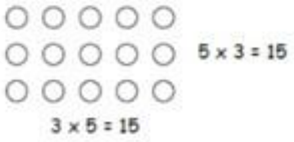
Repeated addition (repeated aggregation)
3 times 5 is 5 + 5 + 5 = 15 or 5 lots of 3 or 5 x 3
Children learn that repeated addition can be shown on a number line.



Children learn that repeated addition can be shown on a variety of manipulatives such as bead strings, numicon,and counters.



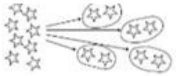
Arrays
Children learn to model a multiplication calculation using an array.



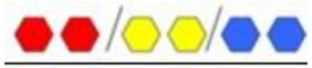
Division

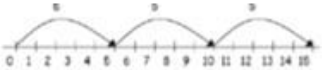
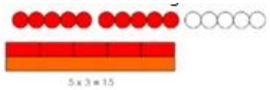
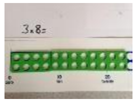
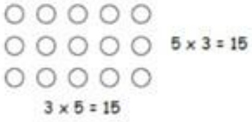
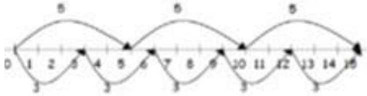
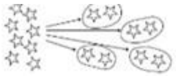
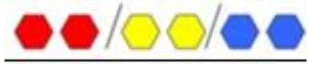
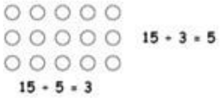
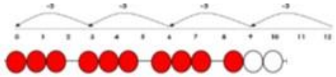
Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s.

Sharing equally
6 sweets get shared between 2 people. How many sweets do they each get? A bottle of fizzy drink shared equally between 4 glasses.



Grouping or repeated subtraction
There are 6 sweets. How many people can have 2 sweets each?



Year 2	Multiplication		Division	
Counts in steps of 2, 3, and 5 from 0, and in tens from any number, forward or backward These concepts are reinforced during Big Maths lessons, where children learn written methods before progressing to fully mental methods.	Children will continue to have real, practical experiences of handling equal groups of objects and counting in 2s,3s, 10s and 5s. Children work on practical problem solving activities involving equal sets or groups.  **Repeated addition (repeated aggregation)** 3 times 5 is 5 + 5 + 5 = 15 or 5 lots of 3 or 5 x 3 Children learn that repeated addition can be shown on a number line.  Children learn that repeated addition can be shown on a variety of manipulatives such as bead strings, numicon, Cuisenaire Rodsand counters.   **Arrays** Children learn to model a multiplication calculation using an array. This model supports their understanding of **commutativity** and the development of the grid in a written method. It also supports the finding of factors of a number 	**Commutativity** Children learn that 3 x 5 has the same total as 5 x 3. This can also be shown on the number line. 3 x 5 = 15 5 x 3 = 15  **Inverse operations** Fact families are introduced to model the 4 related multiplication and division facts within Big Maths. Children learn to state the 4 related facts. 3 x 4 = 12 4 x 3 = 12 12 ÷ 3 = 4 12 ÷ 4 = 3	Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s.  **Sharing equally** 6 sweets get shared between 2 people. How many sweets do they each get? A bottle of fizzy drink shared equally between 4 glasses.  **Grouping or repeated subtraction** There are 6 sweets. How many people can have 2 sweets each?  **Arrays** Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the 'bus stop method' in a written method. This model also connects division to **finding fractions** of discrete quantities.  **Inverse operations** Fact families are introduced to model the 4 related multiplication and division facts within Big Maths. Children learn to state the 4 related facts. 3 x 4 = 12 4 x 3 = 12 12 ÷ 3 = 4 12 ÷ 4 = 3	**Repeated subtraction using a bead string or number line** 12 ÷ 3 = 4  The bead string helps children with interpreting division calculations, recognising that 12 ÷ 3 can be seen as 'how many 3s make 12?' Cuisenaire Rods and Numicon can also help children to interpret division calculations. 

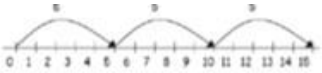
Year 3

From 0 in multiples of 4, 8, 50 and 100

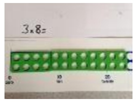
These concepts are reinforced during Big Maths lessons, where children learn written methods before progressing to fully mental methods.

Multiplication

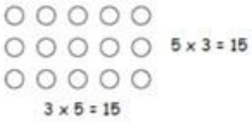
Repeated addition (repeated aggregation)
3 times 5 is 5 + 5 + 5 = 15 or 5 lots of 3 or 5 x 3
Children learn that repeated addition can be shown on a number line.



Children learn that repeated addition can be shown on a variety of manipulatives such as bead strings, numicon, Cuisenaire Rods and counters.



Arrays
Children learn to model a multiplication calculation using an array. This model supports their understanding of **commutativity** and the development of the grid in a written method. It also supports the finding of factors of a number



Commutativity
Children learn that 3 x 5 has the same total as 5 x 3.
This can also be shown on the number line.
3 x 5 = 15
5 x 3 = 15

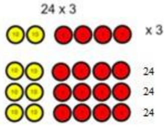
Inverse operations
Fact families are introduced to model the 4 related multiplication and division facts within Big Maths.
Children learn to state the 4 related facts.
3 x 4 = 12
4 x 3 = 12
12 ÷ 3 = 4
12 ÷ 4 = 3

Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.
☐ x 5 = 20   3 x   Δ = 18   O x☐= 32
24 ÷ 2 = ☐   15 ÷ O = 3   Δ ÷ 10 = 8

Scaling
This is an extension of augmentation in addition, except, with multiplication, we increase the quantity by a scale factor not by a fixed amount. For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you will end up with 15 marbles.
This can be written as:
1 + 1 + 1 = 3 scaled up by 5 5 + 5 + 5 = 15

For example, find a ribbon that is 4 times as long as the blue ribbon.

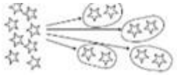
Arrays leading into the grid method
Children continue to use arrays and partitioning, where appropriate, to prepare them for the grid method of multiplication.
Arrays can be represented as 'grids' in a shorthand version and by using place value counters to show multiples of ten, hundred etc



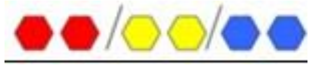
leading to a fully written grid method.

Division

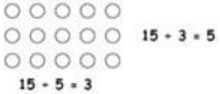
Sharing equally
6 sweets get shared between 2 people. How many sweets do they each get? A bottle of fizzy drink shared equally between 4 glasses.



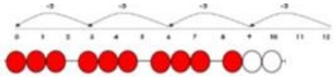
Grouping or repeated subtraction
There are 6 sweets. How many people can have 2 sweets each?



Arrays
Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the 'bus stop method' in a written method. This model also connects division to **finding fractions** of discrete quantities.



Repeated subtraction using a bead string or number line
12 ÷ 3 = 4

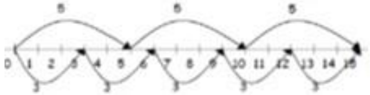
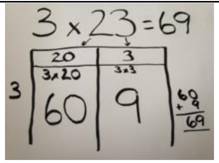


The bead string helps children with interpreting division calculations, recognising that 12 ÷ 3 can be seen as 'how many 3s make 12?' Cuisenaire Rods and Numicon can also help children to interpret division calculations.

Inverse operations
Fact families are introduced to model the 4 related multiplication and division facts within Big Maths.
Children learn to state the 4 related facts.
3 x 4 = 12
4 x 3 = 12
12 ÷ 3 = 4
12 ÷ 4 = 3

Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.
☐ x 5 = 20   3 x Δ = 18   O x ☐ = 32
24 ÷ 2 = ☐   15 ÷ O = 3   Δ ÷ 10 = 8

Commutativity
Children learn that division is **not** commutative and link this to subtraction.

	5 5 5 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 3 3 3 3 3	3 x 23 = 69 20 3 3 3x20 3x3 60 9 60 + 9 69		

Years 4

Counts fluently forwards and backwards to include:
- multiples of 6, 7, 9, 25 and 1000

These concepts are reinforced during Big Maths lessons, where children learn written methods before progressing to fully mental methods.

Multiplication

Inverse operations
Fact families are introduced to model the 4 related multiplication and division facts within Big Maths. Children learn to state the 4 related facts.
3 x 4 = 12
4 x 3 = 12
12 ÷ 3 = 4
12 ÷ 4 = 3

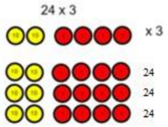
Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

☐ x 5 = 20 3 x Δ = 18 O x = 32 Δ ÷ 10 = 8
24 ÷ 2 = ☐ 15 ÷ O = 3

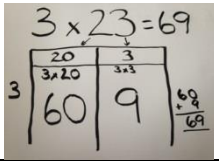
Scaling
This is an extension of augmentation in addition, except, with multiplication, we increase the quantity by a scale factor not by a fixed amount. For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you will end up with 15 marbles.
This can be written as:
1 + 1 + 1 = 3 scaled up by 5 5 + 5 + 5 = 15

For example, find a ribbon that is 4 times as long as the blue ribbon.

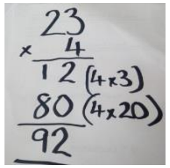
Arrays leading into the grid method
Children continue to use arrays and partitioning, where appropriate, to prepare them for the grid method of multiplication.
Arrays can be represented as 'grids' in a shorthand version and by using place value counters to show multiples of ten, hundred etc



leading to a fully written grid method involving TO x O moving to HTO X O and TO X TO



Short multiplication (columnar) Once children have demonstrated understanding and proficiency with the grid method they will be introduced to short multiplication. Starting with an expanded form.



leading to :

24 × 6 becomes

$$\begin{array}{r} 2\ 4 \\ \times\quad 6 \\ \hline 1\ 4\ 4 \\ \hline {}_{2} \end{array}$$

Answer: 144

Gradation of difficulty (short multiplication)

1. TO x O no exchange
2. TO x O extra digit in the answer
3. TO x O with exchange of ones into tens
4. HTO x O no exchange
5. HTO x O with exchange of ones into tens
6. HTO x O with exchange of tens into hundreds
7. HTO x O with exchange of ones into tens and tens into hundreds
8. As 4-7 but with greater number digits x O

Long multiplication—multiplying by more than one digit
Children will refer back to grid method by using place value counters or Base 10 equipment with no exchange and using synchronised modelling of

Division

Inverse operations
Fact families are introduced to model the 4 related multiplication and division facts within Big Maths. Children learn to state the 4 related facts.
3 x 4 = 12
4 x 3 = 12
12 ÷ 3 = 4
12 ÷ 4 = 3

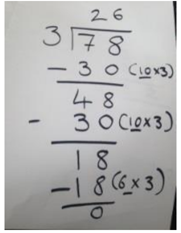
Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

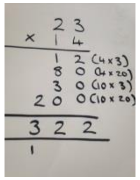
☐ x 5 = 20 3 x Δ = 18 O x = 32
24 ÷ 2 = ☐ 15 ÷ O = 3 Δ ÷ 10 = 8

Commutativity

Children learn that division is **not** commutative and link this to subtraction.

The vertical method- 'chunking' leading to long division
See above for example of how this can be modelled as an array using place value counters. 78 ÷ 3 =



		written recording as a long multiplication model before moving to TO x TO etc.  		

Year 5

Multiplies and divides numbers mentally using known facts and uses derived facts *e.g. 2.3 x 4 = 9.2*
Multiplies and divides whole numbers and those involving decimals by 10, 100 and 1000

These concepts are reinforced during Big Maths lessons, where children learn written methods before progressing to fully mental methods.

Multiplication

Inverse operations

Fact families are introduced to model the 4 related multiplication and division facts within Big Maths. Children learn to state the 4 related facts.

3 x 4 = 12
4 x 3 = 12
12 ÷ 3 = 4
12 ÷ 4 = 3

Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

□ x 5 = 20   3 x Δ = 18   O x □ = 32   Δ ÷ 10 = 8
24 ÷ 2 = □   15 ÷ O = 3

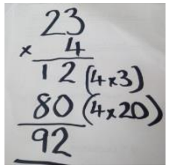
Scaling

This is an extension of augmentation in addition, except, with multiplication, we increase the quantity by a scale factor not by a fixed amount. For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you will end up with 15 marbles.
This can be written as:

1 + 1 + 1 = 3 scaled up by 5 □ 5 + 5 + 5 = 15

For example, find a ribbon that is 4 times as long as the blue ribbon.

Short multiplication (columnar)



leading to :

342 × 7 becomes

$$\begin{array}{r} 3\ 4\ 2 \\ \times\quad\ \ 7 \\ \hline 2\ 3\ 9\ 4 \\ \hline {}_{2\ \ 1}\ \ \end{array}$$

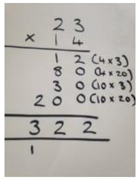
Answer: 2394

Gradation of difficulty (short multiplication)

1. TO x O no exchange
2. TO x O extra digit in the answer
3. TO x O with exchange of ones into tens
4. HTO x O no exchange
5. HTO x O with exchange of ones into tens
6. HTO x O with exchange of tens into hundreds
7. HTO x O with exchange of ones into tens and tens into hundreds
8. As 4-7 but with greater number digits x O
9. O.t x O no exchange
10. O.t with exchange of tenths to ones
11. As 9 - 10 but with greater number of digits which may include a range of decimal places x O

Long multiplication—multiplying by more than one digit

Children will refer back to grid method by using place value counters or Base 10 equipment with no exchange and using synchronised modelling of written recording as a long multiplication model before moving to TO x TO etc.



Leading to:

Division

Inverse operations

Fact families are introduced to model the 4 related multiplication and division facts within Big Maths. Children learn to state the 4 related facts.

3 x 4 = 12
4 x 3 = 12
12 ÷ 3 = 4
12 ÷ 4 = 3

Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

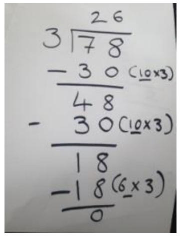
□ x 5 = 20   3 x Δ = 18   O x □ = 32
24 ÷ 2 = □   15 ÷ O = 3   Δ ÷ 10 = 8

Commutativity

Children learn that division is **not** commutative and link this to subtraction.

The vertical method- 'chunking' leading to long division

See above for example of how this can be modelled as an array using place value counters. 78 ÷ 3 =



Short division

98 ÷ 7 becomes

$$\begin{array}{r|l} & 1\ \ 4 \\ \hline 7 & 9\ {}^{2}8 \end{array}$$

Answer: 14

Gradation of difficulty (short division)

1. TO ÷ O no exchange no remainder
2. TO ÷ O no exchange with remainder
3. TO ÷ O with exchange no remainder
4. TO ÷ O with exchange, with remainder
5. Zero in the quotient e.g. 816 ÷ 4 = **204**

Dealing with remainders

Remainders should be given as integers, but children need to be able to decide what to do after division, such as rounding up or down accordingly. e.g.:

- I have 62p. How many 8p sweets can I buy?
- Apples are packed in boxes of 8. There are 86 apples. How many boxes are needed?

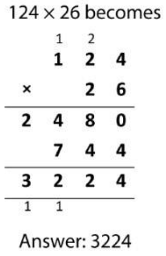
Gradation of difficulty for expressing remainders

1. Whole number remainder

432 ÷ 5 becomes

$$\begin{array}{r|l} & 8\ \ 6\ \ \mathbf{r\,2} \\ \hline 5 & 4\ \ 3\ {}^{3}2 \end{array}$$

Answer: 86 remainder 2

		 124 × 26 becomes $\begin{array}{cccc} & {\scriptstyle 1} & {\scriptstyle 2} & \\ & 1 & 2 & 4 \\ \times & & 2 & 6 \\ \hline 2 & 4 & 8 & 0 \\ & 7 & 4 & 4 \\ \hline 3 & 2 & 2 & 4 \\ \hline {\scriptstyle 1} & {\scriptstyle 1} & & \end{array}$ Answer: 3224	6. As 1-5 HTO ÷ O 7. As 1-5 greater number of digits ÷ O 8. As 1-5 with a decimal dividend e.g. 7.5 ÷ 5 or 0.12 ÷ 3	

Year 6

Continues to use all known facts to calculate mathematical statements with increasing complexity

These concepts are reinforced during Big Maths lessons, where children learn written methods before progressing to fully mental methods.

Multiplication

Inverse operations
Fact families are introduced to model the 4 related multiplication and division facts within Big Maths. Children learn to state the 4 related facts.

3 x 4 = 12
4 x 3 = 12
12 ÷ 3 = 4
12 ÷ 4 = 3

Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

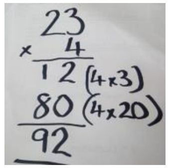
□ x 5 = 20   3 xΔ = 18   O x = □32 Δ ÷
24 ÷ 2 = □   15 ÷ O = 3   10 = 8

Scaling
This is an extension of augmentation in addition, except, with multiplication, we increase the quantity by a scale factor not by a fixed amount. For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you will end up with 15 marbles.
This can be written as:

1 + 1 + 1□= 3 scaled up by 5□5 + 5 + 5 = 15

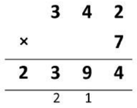
For example, find a ribbon that is 4 times as long as the blue ribbon.

Short multiplication (columnar)



leading to :

342 × 7 becomes



Answer: 2394

Gradation of difficulty (short multiplication)

1. TO x O no exchange
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4. HTO x O no exchange
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6. HTO x O with exchange of tens into hundreds
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Long multiplication—multiplying by more than one digit
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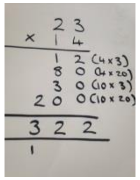

Leading to:

Division

Inverse operations
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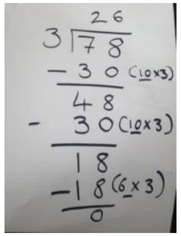
3 x 4 = 12
4 x 3 = 12
12 ÷ 3 = 4
12 ÷ 4 = 3

Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

□ x 5 = 20   3 x Δ = 18   O x = □32
24 ÷ 2 = □   15 ÷ O = 3   Δ ÷ 10 = 8

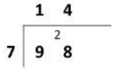
Commutativity
Children learn that division is **not** commutative and link this to subtraction.

The vertical method- 'chunking' leading to long division
See above for example of how this can be modelled as an array using place value counters. 78 ÷ 3 =



Short division

98 ÷ 7 becomes



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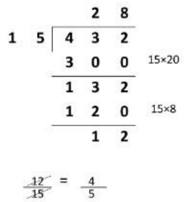
3. TO ÷ O with exchange no remainder
4. TO ÷ O with exchange, with remainder
5. Zero in the quotient e.g. 816 ÷ 4 = **204**

Long division —dividing by more than one digit
Building upon their knowledge of chunking children will:

432 ÷ 15 becomes



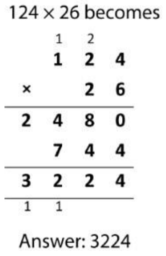
Dealing with remainders

Remainders should be given as integers, but children need to be able to decide what to do after division, such as rounding up or down accordingly. e.g.:

- I have 62p. How many 8p sweets can I buy?
- Apples are packed in boxes of 8. There are 86 apples. How many boxes are needed?

Gradation of difficulty for expressing remainders

1. Whole number remainder
2. Remainder expressed as a fraction of the divisor
3. Remainder expressed as a simplified fraction
4. Remainder expressed as a decimal

		 124 × 26 becomes $\begin{array}{cccc} & {}^{1} & {}^{2} & \\ & 1 & 2 & 4 \\ \times & & 2 & 6 \\ \hline 2 & 4 & 8 & 0 \\ & 7 & 4 & 4 \\ \hline 3 & 2 & 2 & 4 \\ \hline {}_{1} & {}_{1} & & \end{array}$ Answer: 3224	6. As 1-5 HTO ÷ O 7. As 1-5 greater number of digits ÷ O 8. As 1-5 with a decimal dividend e.g. 7.5 ÷ 5 or 0.12 ÷ 3	